Experiment 9. Spring Constant

Is rubber more elastic than steel? The answer lies in the concept of elasticity.

Objective:

To determine the spring constant of a spring by (a) static method, and (b) dynamic method.

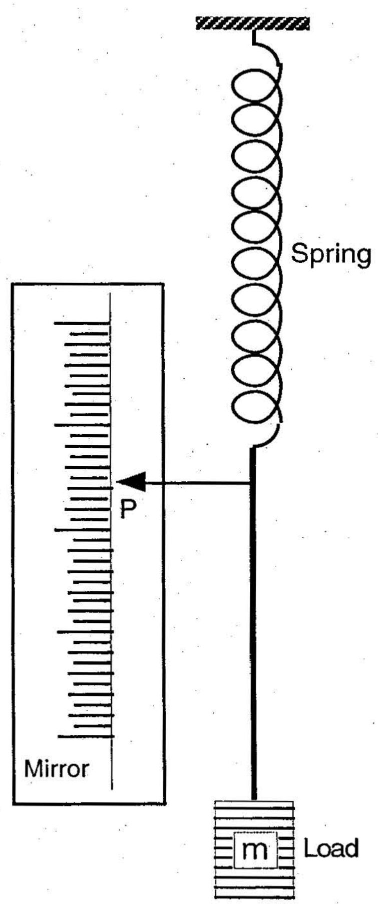
Apparatus:

A spring, a hanger with a light aluminium pointer, a small scale etched on a plane mirror strip, weights, a timer.

Theory:

(a) Static method:

Consider a spring hanging from a rigid support. When a load m is suspended from the free end of the spring, an external, F_{ext}, acts on the spring in the downward direction. The support applies an equal force in the upward direction. Thus the spring has a balanced system of forces acting on it and it is in equilibrium. The length of the string increases and an internal (restoring) force F_{int} is developed in the spring due to the elasticity of the spring. This internal force tends to bring the spring back to its original length when the external forces are withdrawn. Note that F_{int} and F_{ext} are equal in magnitude.



According to Hooke's law,

F_{int} is directly proportional to x, the change in the length of the spring.

Thus $F_{int} = -k\,x$ (1)

Here k is the constant of proportionality, known as the spring constant or force constant. The minus (-) sign indicates that F_{int} and x are in opposite directions.

Obviously,

$$F_{ext} = k\,x. \tag{2}$$

Eqs. (1) and (2) indicate that the spring constant k is numerically equal to the force required to change the length of the spring by 1 unit.

If the load on the spring is M and M_p is the sum of the mass of the hanger and the effective mass of the spring, $F_{ext} = (M+M_p)g$. In this experiment, values of x for a number of values of F_{ext} are determined by changing M. Then a graph of M vs. x is plotted and k is determined from the slope of the graph.

(b) Dynamic method:

If a mass M is suspended from a spring of spring constant k and the system is made to oscillate, then for small amplitudes of oscillations (that is, within elastic limit) along the length of the spring, the motion of the system is simple harmonic. In this case, the time period T of the system is given by

$$T = 2\pi\sqrt{\frac{M + M_p}{k}}, \qquad (3)$$

where M_p is the effective mass of the spring which is approximately equal to 1/3 the mass of the spring (Reference: Vibrations and Waves by A. P. French, pages 60-61).

In this experiment, M_p is eliminated in the following manner:

Eq. (3) gives $T^2 = \frac{4\pi^2}{k} M + \frac{4\pi^2}{k} M_p$.

Thus if we plot T^2 vs. M, then for a given spring (k and M_p constant), the graph will be a straight line. By choosing two points (M_1, T_1^2) and (M_2, T_2^2) on the graph, we get

$$T_1^2 = \frac{4\pi^2}{k} M_1 + \frac{4\pi^2}{k} M_p \text{ and } T_2^2 = \frac{4\pi^2}{k} M_2 + \frac{4\pi^2}{k} M_p.$$

By subtracting one equation from the other, we get

$$T_2^2 - T_1^2 = \frac{4\pi^2}{k}(M_2 - M_1),$$

$$\text{or } k = 4\pi^2\left(\frac{M_2 - M_1}{T_2^2 - T_1^2}\right). \qquad (4)$$

Procedure:

(a) Static method:

1. Arrange the apparatus as shown in the figure. Make sure that the aluminum pointer is close to the mirror on which the scale is etched but it does not touch the mirror.

2. Read the position of the pointer on the scale. Avoid the parallax error. This is done by reading the position of the pointer on the scale when the image of the pointer is hidden behind the pointer. (Remember the procedure of the experiment on equilibrium.)
3. Increase the load on the spring in equal steps and read the position of the pointer each time. Thus fill out the first two columns of the first table of the data sheet.
4. Now gently pull the load down through about 0.5 cm. Release it gently and let it come to rest. Take the reading of the position of the pointer and enter it in the last row of column 3 of the table. Gently decrease the load in equal steps and thus fill out the rest of column 3 of the table.

(a) Dynamic method:

5. Place a suitable load on the hanger and start the oscillations of the system. Make sure that the oscillations are along the length of the spring and the amplitude is small. Find the time for 40 (or 50) oscillations 3 times.
6. Repeat the procedure by changing the load on the spring in equal steps.

York College of The City University of New York

Physics I Name:

Experiment No. 9: Pre-Lab Questionnaire

1. A student finds the following values from a graph between M and x in the static method of determining the spring constant k:

   x_1 = 0.5 cm; x_2 = 7.8 cm; M_1 = 120 gm; M_2 = 300 gm.

   Calculate the value of k.

2. Consider the following data for an experiment to study the spring constant by dynamic method:

   Number of oscillations, N = 25

   Load M_1 = 120 gm; average time for N oscillations = 11.0 seconds

   Load M_2 = 280 gm; average time for N oscillations = 16.9 seconds

   Calculate the spring constant k.

   (Note that the mass of the hanger is not given.)

h
b

Experiment No. 9

Name:	Marks:
Partner:	Remarks:
Section:	
Date Submitted:	

Title:

Objective:

Theory/Formulas:

Data Sheet

(a) Static method:

No.	Load M on the hanger	Position of the pointer (x)		Average x
		Load increasing	Load decreasing	
1				
2				
3				
4				
5				
6				

Plot a graph between M and average x.
From the graph,
$M_1 =$; $x_1 =$; $M_2 =$; $x_2 =$

k =

(b) Dynamic method:
Number of oscillations, N =

No.	Load M	Time for N oscillations				Period T	T^2
		R_1	R_2	R_3	Average R		
1							
2							
3							
4							
5							
6							

Plot a graph between M and T^2.
From the graph,
$M_1 =$; $T_1{}^2 =$; $M_2 =$; $T_2{}^2 =$

k =

5
6

Experiment No. 9: Questions

1. State Hooke's law.

2. What is meant by the spring constant of a spring? (Remember, it is also known as force constant.)

3. Under what conditions the period of oscillations of a mass on a spring is given by

$$T = 2\pi \sqrt{\frac{M}{k}} \ ?$$

4. Why is it not necessary to use the mass of the hanger in the calculations of the load on the spring?

5. If the load M on the spring is made 4 times its previous value, will the time period then become exactly double its previous value? Explain your answer.